U. S. ARMY MATERIEL COMMAND

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LEARNING CURVE METHODOLOGY FOR COST ANALYSTS

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HEADQUARTERS

U.S. ARMY MATERIEL COMMAND

LEARNING CURVE METHODOLOGY

for

COST ANALYSTS

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PREFACE

Cost estimates are sometimes poorly prepared because the learning curve is misunderstood or badly applied.

The learning curve as a tool, regression analysis as a curve fitting technique and various devices for adjusting learning data from practical process knowledge are addressed separately in a voluminous literature. This paper puts down the main framework of necessary concepts in the application of the learning curve with appropriate references to the literature.

Experience shows that, at some point in an analysis, the estimator is required to enter opinions because of lack of data, incomplete knowledge of the process or other causes beyond his control. The emphasis in this paper is to distinguish between mathematics and judgment; between calculation and intuition; putting cautions on the analyst to provide the reviewer with visibility as to where one ends and the other begins.

Topics include a description of the forms of the learning curve with distinctions among possible variables, various necessary calculations and conversions, fundamental concepts related to the location of a straight line in two dimensional space, factors which contribute to learning in industrial processes and adjustments for special circumstances.

The document was prepared to provide guidance to cost analysts in the US Army Materiel Command.

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1. INTRODUCTION

In estimating costs of manufactured product, the principle of declining costs resulting from extended production runs must usually be considered. Use of such cost-quantity relationships, referred to currently in the defense community as learning curves, * is common among Army estimators and associated industrial contractors.

This paper is designed as a summary survey of concepts and procedures necessary in the application of the learning curve to cost analysis for contract negotiation, programming and budgeting. The principal purpose is to highlight appropriate areas of investigation in the distinction between the intuitive or decision making, and the mathematical and computational functions.

Section 2 describes the learning model and justifies its use in cost-quantity projections. Section 3 introduces concepts which are necessary to the understanding of computational procedures.

Sections 5 to 7 deal with the type of analyses which cost analysts are required to perform.

Section 4 provides the link between the mathematical and the intuitive through geometrical considerations associated with the straight line representation of the learning curve.

Sections are numbered for convenience by a digit; sub-sections by two digits.

For the sake of brevity, mathematical derivations are not given nor are detailed computational procedures. The more important mathematical expressions are listed in the Appendix. The more important references are noted in the text and listed in the Reference List.

2. THE LEARNING MODEL

The concept introduced by Wright and Crawford, early investigators in learning curve applications, is that, as the total quantity of units produced doubles, the cost per unit declines by some constant percentage. One form treats the cost per unit as the average cost of a given number of units, the other as the cost of a specific unit.

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*Also - experience, improvement, cost-reduction and progress curves

2.1 Description

The unit curve always lies below the cumulative average curve. The distinction between these curves should be kept clear.

When the cost per unit is assumed to be the average cost over cumulative units produced, the form of the learning curve is known as the log-linear cumulative average curve; when the cost per unit is treated as the cost of a specific unit, it is known as the log-linear unit curve. Either form is convertible to the other by the use of appropriate tables.[1/] The terms are usually abbreviated to cumulative average curve and unit curve; the former is sometimes referred to as the Wright formulation and the latter as the Crawford formulation.

When the unit curve on logarithmic grids is a straight line as shown in Figure 1, the cumulative average curve is a curved line asymptotic to a straight line of the same slope lying above the unit curve.[2/] The latter is the more popular in the Army and is the formulation that will be discussed in the remainder of this paper.

When the cumulative average curve on logarithmic grids is a straight line, the unit curve is a curved line asymptotic to a straight line of the same slope lying below the cumulative average curve.

Although the second formulation is not shown in an illustration, it is obvious that there would be a difference in all costs except the first unit cost.

2.2 Applicability

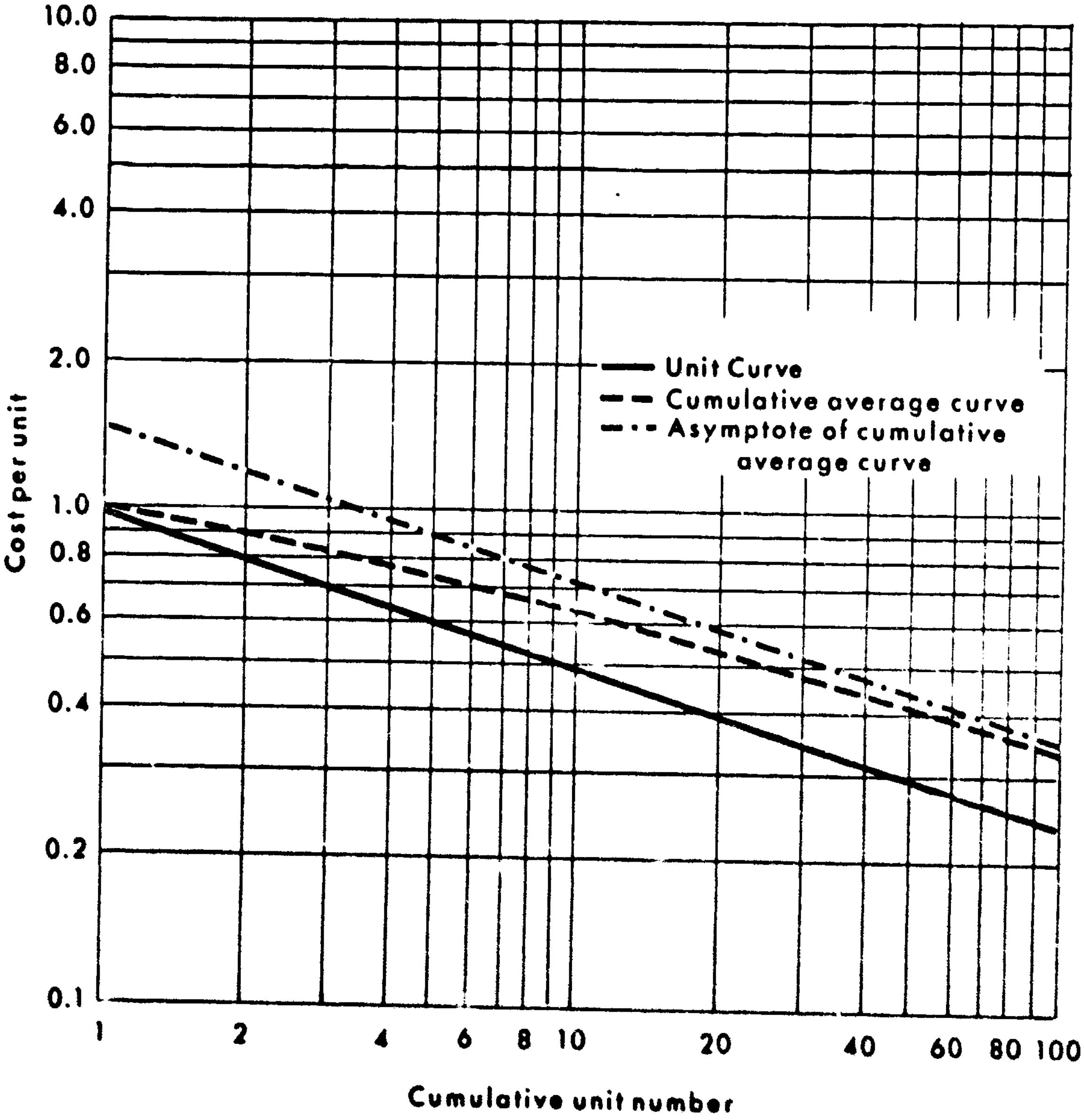
The fundamental concept of the learning curve as applied to the manufacture of product appears to be a reasonable approach to cost-quantity projections. The model has been tested on data from the aircraft industry where it fits out to thousands of units produced. There is a growing list of reports and studies that indicate applicability of this curve to other types of industrial processes.[3/4/] Supporting the learning concept is the estimate of the Bureau of Labor Statistics that the national productivity increase in the period 1960-6 rose at the rate of 3.2% per year.

1/ Experience Curve Tables, page 16

2/ Asher, pages 21-23. Equations which relate these curves.

3/ Hartmeyer, Section 22

4/ Hirschmann; Applicability to petroleum, electric power and steel production; pages 133-135.



COMPARISON OF LEARNING CURVES RESULTING FROM THE ASSUMPTION OF A LINEAR UNIT CURVE ON LOGARITHMIC GRIDS

Figure 1

There is some doubt, however, as to the applicability of the model at high values of cumulative units in airframes, such as in the region above 3000 units, and in the small machined parts industries generally.

The principal applications of the learning curve assumption appear in those processes that include assembly operations or a mix of assembly and machining operations. Machine paced operations, such as the production of machined parts (screws, engine blocks, gears, etc.), and carefully engineered manual processes, such as the assembly of small electronic parts on a printed circuit, show little or no learning. The factor by which the unit cost reduces as cumulative units produced increases is influenced by the type of industrial process.

2.3 Data Collection and Uncertainty

Difficulties associated with applications of the learning curve are related to the determination of the parameters that shape the curve and to the collection and verification of data inputs into the fundamental equation. Data is available from the Defense Contractors Planning Reports (DCPR), Aircraft Manufacturing Planning Reports (AMPR), Missile Manufacturing Planning Reports (MMPR), contract files and other sources. In some cases, these data go back many years, but difficulties are experienced in achieving comparability. These relate to the amount of indirect labor or costs which were included; to the unit at which a model change occurred; to the amount and type of costs associated with similar commercial or other service procurement; etc.

Existing reporting structures presently applicable to defense aircraft and missile procurement under the Cost Information Reports (CIR) are designed to correct these data collection difficulties. It will be some years, however, before the full impact of improvements in data collection will be felt in the development of the learning curve parameters and in the development of cost estimating relationships generally.

3. CALCULATIONS & CONVERSIONS

3.1 Fundamental Relationship

The concept expressed by Crawford may be written as a two variable relationship connecting the dependent variable in direct labor hours or direct labor costs with cumulative units produced. The right hand member is the product of the theoretical first unit cost and cumulative units produced raised to a power. (Equations throughout will be listed in the Appendix.) The exponent b is a small

negative fraction; for example - for an 80% slope, b is -0.3219, which means that the value of the dependent variable reduces by some fraction to the next succeeding unit.

If logarithms are taken on each side of the equation the log-linear form of the learning curve is expressed.*

3.2 Power Expressed as a Slope

Practitioners find it more convenient to deal with the graphs of the equation and to express the slope as a percent; the slope being the complement of the constant percentage reduction p in item cost, occurring with doubled production.**

Slope and the corresponding exponent can be related by an equation and by a tabular conversion.***

The parameters a and b are determined by statistical regression analysis techniques. For descriptive purposes, the value of a may be considered as the point on the y axis (on y for $x=1$ unit) which is determined from a set of scattered points. The line is "backed up" to the axis, i.e. the y intercept.

3.3 Lot Mid-Points

The log-linear curve for an 80% slope is illustrated in Figure 2.

Units cost are seldom available for a specific unit of production. Rather cost or manhour data is available over a production lot.

Figure 3 again illustrates the 80% curve, theoretical first unit cost of 10, but shown on arithmetic scalings. The arithmetic scaling is convenient in gaining an appreciation of the function under consideration.

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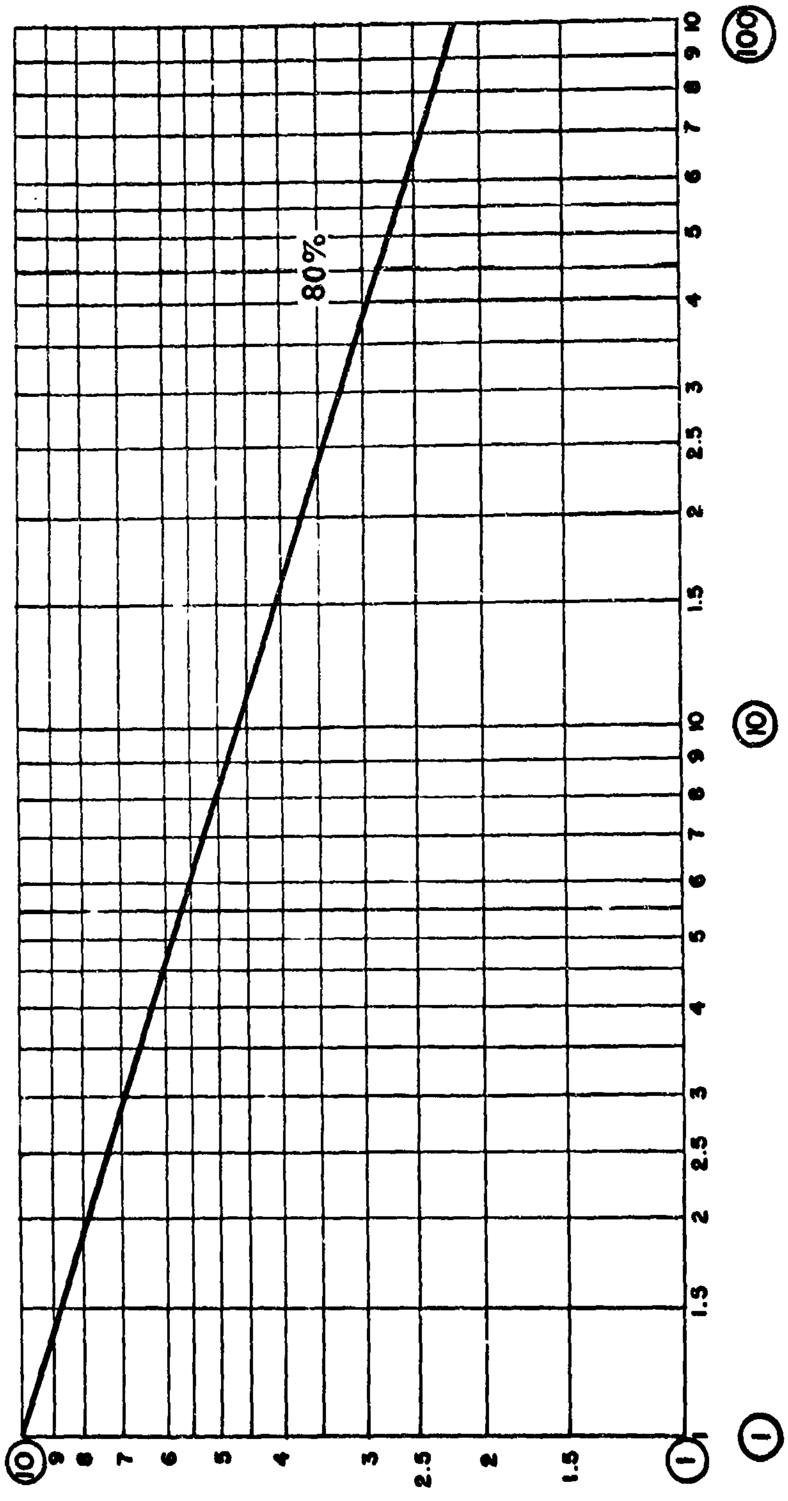
*See (2), Appendix.

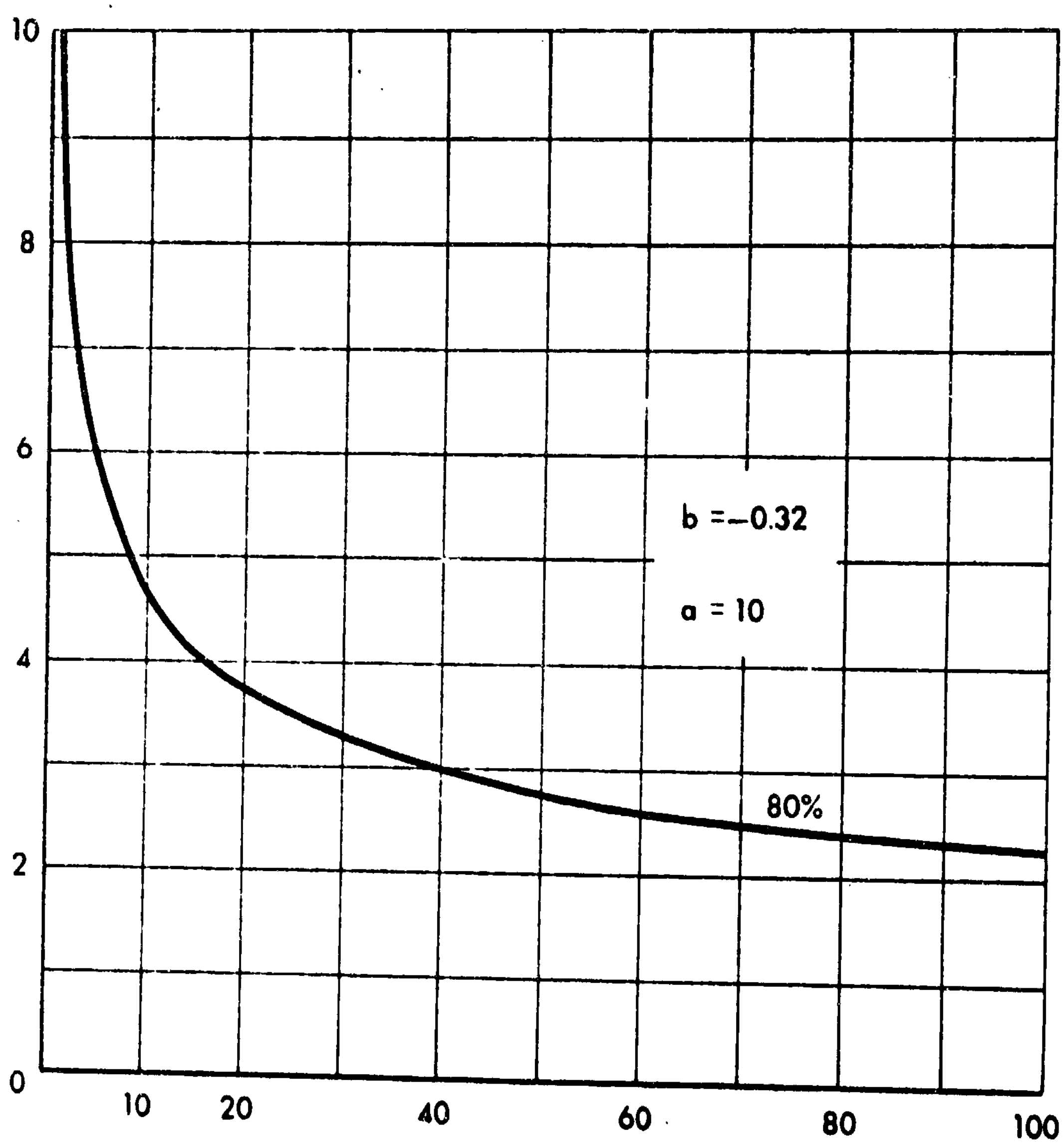
**Thus b is related to the constant percentage reduction p as follows: $b = \log(1-p)/\log 2$; because by definition of the learning curve $a(2x)^b$ must equal $(1-p)ax^b$, or $2^b = 1-p$. The quantity $(1-p)$ is conventionally referred to as the slope. A slope of 80% signifies a constant percentage reduction of 20%, a slope of 90% signifies a reduction of 10%.

***Equation #3 in the Appendix. Table I, Experience Curve Tables.



LEARNING CURVE - 80% ON LOGARITHMIC SCALES

Figure 2



LEARNING CURVE - 80% ON ARITHMETIC SCALES

Figure 3

The arithmetic lot mid-point is determined by adding the unit number of the first unit in a lot to the unit number of the last unit of the lot and dividing by two. Obviously, plotting the average of the first and the last units of a lot at the arithmetic lot mid-point would give an erroneous plot point because of the shape of the curve.

The Experience Curve Tables include tables and a procedure for determining the algebraic lot mid-point.1/

Additionally the algebraic lot mid-point may be determined by formula.*

Nevertheless, the procedure described above as the arithmetic lot mid-point is used for approximate plotting of points on logarithmic grids. From Figure 3 it may be observed that where x is large, although the curve is not a straight line, it is almost a straight line. This is particularly true for small increments of x.

In early lots, such as in the range 1-50, the curve changes rapidly. For this reason many authorities recommend a rule of thumb correction known as the "one third rule".5/6/7/

The rule is to plot the average cost of the first lot at the unit point determined by dividing the number of units in the lot by three. In all subsequent lots, the average lot cost is plotted at the unit point determined by dividing the number of units in the lot by two.

Data is available ususally in the form of a total cost over a lot which is a cumulative total cost.** The average cost is determined by dividing the total cost of a lot by the number of units in the lot. This is the value which is plotted as a cost on the logarithmic grids against the arithmetic lot mid-points in the simplified approximate procedure above.

1/ The tables are arranged by slope for the first unit cost of one. For each value of x; unit cost, cumulative cost and cumulative average cost over x=1 to x=n are given. The procedure is described on page 20.

5/ Jordan, pages 3-10.

6/ DCAA Manual, Appendix F, Section F-302

7/ Dahlhaus, page 10.

* Equation 4 in the Appendix.

** Would be represented by an area under the curve of Figure 3; a definite integral.

If approximate methods are used, they should be identified as such. The analyst should note the fact that arithmetic lot midpoints were used if such is the case.

4. LOG-LINEAR LEARNING CURVE ANALYSIS

Analysts who either prepare learning data or who review learning data sometimes overlook the fact that mathematics which is based on preference or whim will not create good analyses. The model described is at best only an approximation of the real world. The logarithmic formulation of the learning curve makes it subject to all the rules that pertain to the location of a linear relationship on arithmetic scalings.

4.1 Geometry of the Straight Line

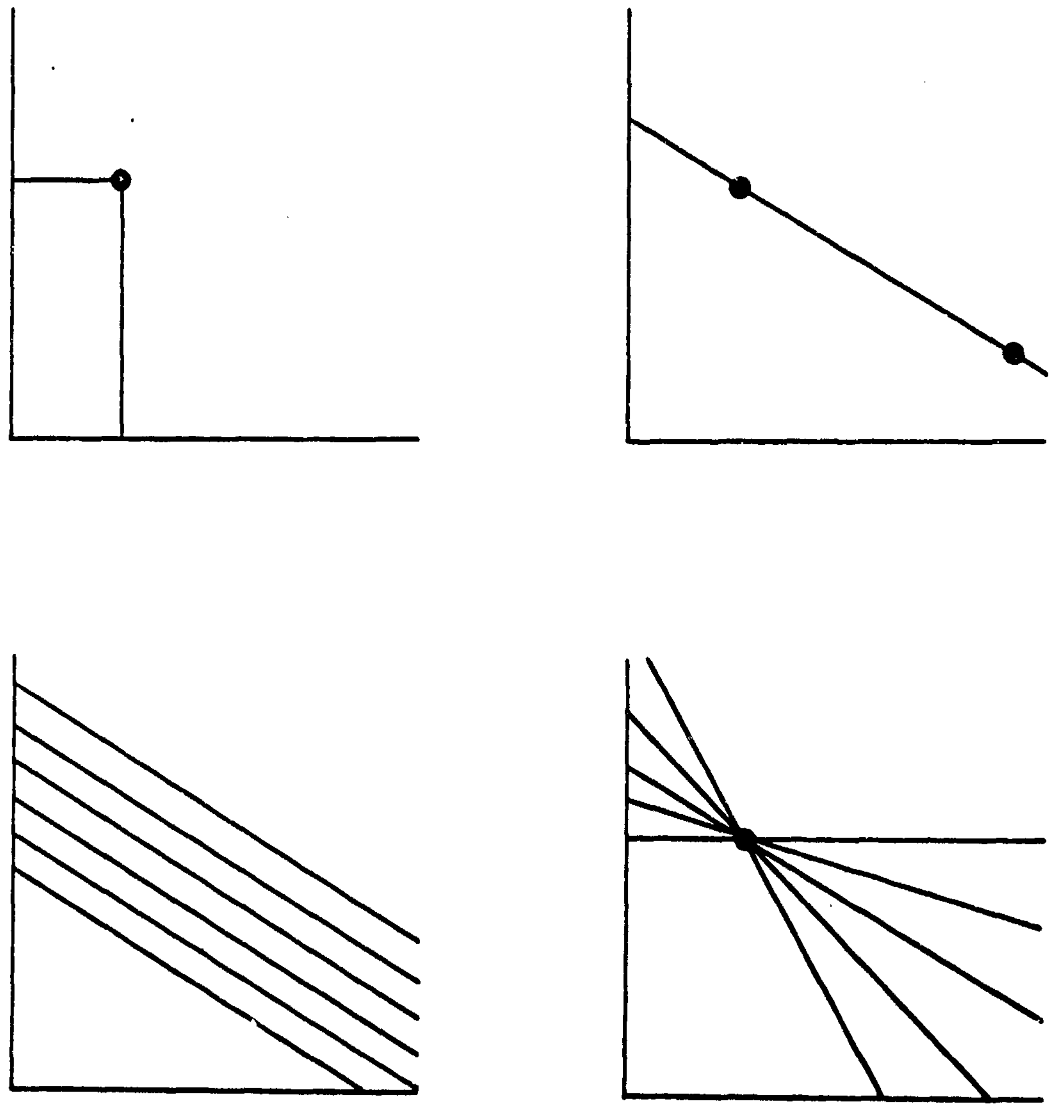
A straight line is fixed in two dimensional space if two points are determined or if one point and the slope is determined. To locate a point in the Cartesian system of coordinates, two values are required; one value measured along each axis. If only the slope is known, there are an infinite number of possible straight lines. If only one point is known, there are an infinite number of possible straight lines. These principles are illustrated in Figure 4.

Failures to recognize these geometric concepts lie at the root of much of the confusion in cost analysis today. Applications of the learning curve based on the present data base will include many judgments based upon past experience, approximations from conversion factors, assumptions relative to cost, hours, schedule, etc. But judgment, approximation and assumption should be identified clearly so that reviewers and analysts at other organizational levels can determine the basis upon which projections were made. A discussion of the factors that should be considered and discussed relative to these judgments and assumptions is reserved for later sections.

4.2 Regression Analysis

In addition to the previous geometric considerations, a line may be determined by the statistical techniques of regression analysis.

The values resulting from any data collection system should be considered as sampled data , ints. Regression analysis seeks to identify the line of best fit to a set of data points based on a statistical criteria. No one of the data points is known with certainty and it is assumed that all vary around the calculated line.



GEOMETRY OF THE STRAIGHT LINE

Figure 4

Appropriate equations are listed in the Appendix. An ample statistical literature exists [8/] and for this reason no discussion is necessary to justify or explain those equations.

In the interests of simplicity, the equations are shown for a linear relationship. The logarithmic transformation of the learning model creates computational but not conceptual difficulties. The computational substitutions, therefore, which enter after the summation symbols in the Appendix, should be understood as the logarithms of the variables.

A line is often fitted visually to a set of scattered points plotted on logarithmic paper. This is an approximate method required by the pressure of circumstances and the analyst should note the fact that he has used an approximate method; frequently noted on the graph as "eye ball fit".

5. FACTORS CONTRIBUTING TO LEARNING [3/]

The learning process is due to a number of factors. In the narrow sense, a learning curve considers only the individual operator learning the sequence and techniques of his job and making improvements over time and quantity on those sequences and techniques. In the broader sense, this type of learning accounts for only part of the improvement which takes place in a production program. For this reason the broader terms presented in the notes to the introductory paragraph are sometimes used. Some of the more important contributory factors are as follows: Improved methods, processes, tooling, machine and manufacturing designs, management learning and debugging of engineering data.

5.1 Improved Methods

As quantities are increased, more specialized tooling, such as jigs, dies, templates and special purpose machine tools, becomes economical.

The high tooling or engineering cost is offset by the reduced labor per unit. Also, the constant efforts of manufacturing engineering, supervision, and employee suggestion systems cause the

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8/ Ezekiel & Fox and others.

3/ A large portion of this section is taken almost verbatim from Section 22 "Electronics Industry Cost Estimating Data", by Fred C. Hartmeyer, the Ronald Press Co., 1964.

productivity of labor to increase even on operations that were once considered to be performing at maximum efficiency.

5.2 Management Learning

Most men in the manufacturing process from operator to plant manager learn the idiosyncrasies of a new production unit as it affects their jobs. The net result, after the initial learning process, is a smoother flowing management system that both supports and guides the basic production effort causing eliminations of material shortages, improvement of plant layouts and improvement of paper work systems and procedures (route sheets, operation descriptions, etc.). All activities supporting the direct labor of the production operator or assembler undergo an improvement process as the work progresses.

5.3 Debugging of Engineering Data

The correction of minor dimensional and material errors on engineering drawings is a major task during the placement of a newly designed unit into production. According to the time allotted and the effort applied by both the design engineering and production planning activities, the drawing debugging may have a very little or very great influence on production labor. At best, all errors will be corrected prior to production; at worst, the errors will be discovered and corrected while the unit is in production. The latter typically results in learning curves of the 70 to 75% range, i.e., high initial cost but rapid improvement toward lower costs.

It was previously pointed out that the learning curve is an approximation of the real world, and that judgment intuition and analysis must be applied. No amount of intuition will supplant real world historical cost data; but factors such as the above should be considered in an analysis, and discussion should be directed to these factors in the establishment of the specific slope used.

5.4 Production Processes and Slopes

Slopes vary primarily with newness of product or amount of innovation contained in a product. Slopes become steeper with new product and new methods, and flatter with repetitive and machine paced processes. Complexity of product from the point of view of design difficulty has little effect on the slope. Computers, guidance systems and radars that have a high proportion of machine paced and repetitive operations, (such as wiring transistors onto printed circuits) have remarkably flat slopes. In such processes the product is complex but the operations are comparatively simple, having long cycle times with random repetitive elements.

Lacking any further information, analysts frequently use an old rule of thumb relating the proportion of assembly to machining direct labor hours: 75 assembly to 25 machining, 80% learning; 50 to 50, 85% learning; 25 to 75, 90% learning.4/9/

The controlling factor affecting learning is the total process, not the product. This does not preclude the effects of design changes--such as engineering change orders that have influence in disrupting the process and creating temporary work stoppages and process changes.

6. ENGINEERING & OTHER MAJOR CHANGES

The use of the learning curve to measure a rate of change is a dynamic method of analyzing costs. Where other methods assume a constancy in composition of a product and in the technology of its production, the theory of the learning curve assumes the constancy of change; it assumes that the rate of change is the factor that will be constant. To the extent that this assumption is true, the curve when appropriately plotted will be linear; and the slope of the curve will be constant. Conversely, to the extent that it is not true, the plotted data will form a curvilinear and usually a slightly irregular pattern.

6.1 Effect of Engineering Changes

It must be recognized that these learning curve assumptions encompass only those changes that comprise the normal and continuing pattern of change. There are, however, other changes that occur only occasionally but that have an abrupt and major impact on unit costs. These changes tend to produce a sharp and noticable deviation in the slope and vertical position of the learning curve. As shown between lots 5 and 6 in Curve A of Figure 5, major changes in the design of a product, commonly known as engineering changes, are one of the most common causes of these sharp deviations in the level and slope of the curve. There are, however, a number of other causes which can have the same or a similar effect, such as a major change in tooling and equipment, a major shift towards automation, or the production of a major component previously purchased.

4/ Hirschmann, page 126

9/ Andress, page 89

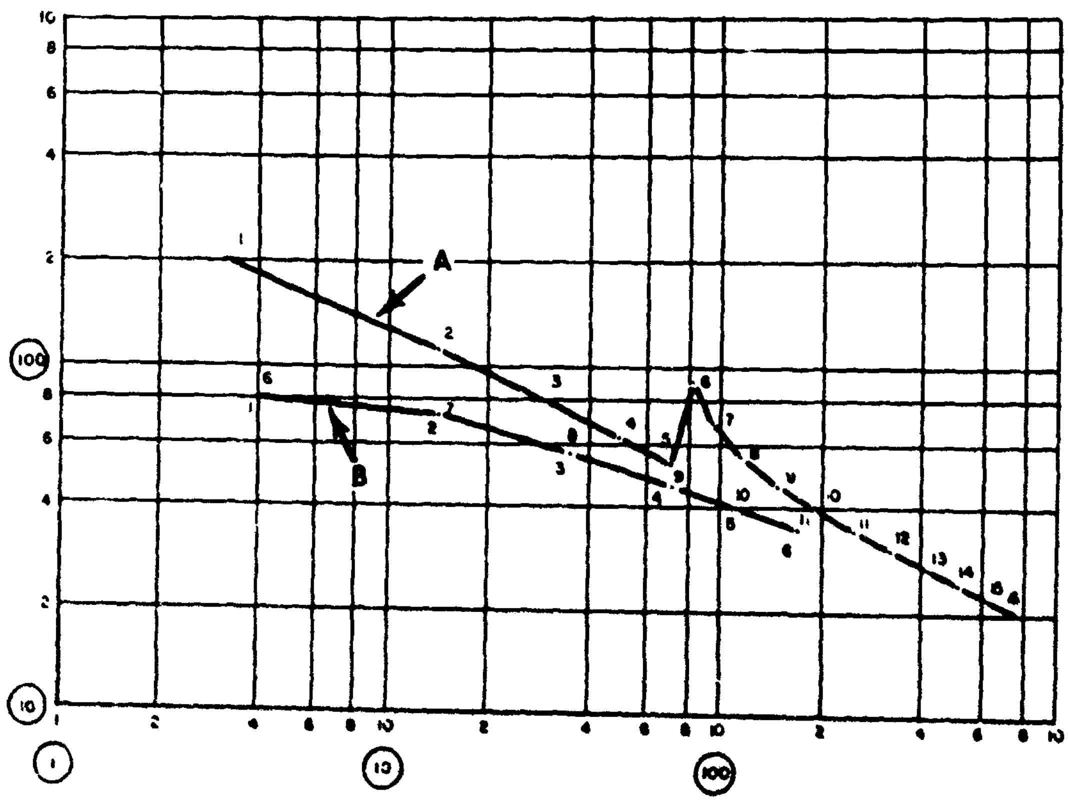

Figure 5

Historically and well after the fact, the impact of engineering and other major changes is relatively simple to analyze. However, the use of data which reflects recent effects of such changes, when a forecast of costs is required, is largely a matter of judgment as to what the level and slope will be and when it will probably stabilize. The difficulty of forecasting from a curve that reflects an engineering or other major change may be seen from the example given in Curve A of Figure 5. Here a major change was made in a component in lot 6. As a result, a sharp rise in the vertical position of the curve occurred between lots 5 and 6. Though the curve slopes back sharply thereafter, it does not begin to reflect a stabilized slope until lot 10. Here it approaches and becomes asymptotic to the slope before the change.

Extrapolation of the basic trend at lot 6 to forecast lots 7 to 10, would be meaningless. Extending any segment created by any adjacent pair of these points would also be meaningless. To overcome these difficulties, techniques have been developed to adjust for the effects of engineering changes. The analyst will seldom, if ever, need the detail and refinement that is characteristic of some methods. The simple procedure described below will be useful in the development of cost estimates.

6.2 <u>Adjustment Procedure</u>

The procedure for adjusting a learning curve to compensate for the effect of engineering and other major changes is known as splicing the curve. A simple graphical method is illustrated in Figure 5.

In this method, the portion of the curve affected by the engineering change is treated as though it pertained to a new product. In effect it does.

The lot in which the change was made is replotted as a new number one lot; and each succeeding lot is replotted as the second, third and succeeding new lot. This creates* Curve B from the data points of Curve A as shown in the figure.

It is the proper function of a cost analyst to decide the slope and level of the learning curve for cost extrapolation. He should, therefore, determine when a change is of such a magnitude to warrant consideration as a new model and to make an adjustment according to the procedure described.

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* Details of this procedure are given in the <u>DCAA Manual</u>, Appendix F, Section F-501.

It is also his responsibility after appropriate discussion with engineers, project managers, on-site auditors and plant representatives to document his reasoning in the extrapolation of learning data.

7. GENERAL CONSIDERATIONS

The cost of the last unit or lot produced may frequently be used as the starting point for the projection of an improvement curve, unless it contains some abnormality or departs significantly in some way from the line fitted to the preceding data. Because it is the latest experience available, such a data point is usually a reliable base for estimating future costs. Note that in this case the analyst is following the analytical procedure of fixed point-fixed slope discussed in Section 4.1 above. However, a recently introduced change may not be reflected in a curve, because new parts are not being used in the finished product.

This suggests that, in some instances, the starting points for forecasting future production costs should be the most recent cost of each operation in the production of each part of each compenent. This would be a composite cost showing what it would cost to produce the product if all work were done today. It is the most recent cost of each production operation that is the best base from which to forecast the cost of any future production.

The cost of products to be manufactured from parts and assemblies already produced is largely a matter for factual determination and not projection. Only a short term projection of assembly costs is required. If there is a volume of such parts and assemblies on hand, the analyst should consider their cost separately from the forecasting of future production costs.

7.1 Detailed Investigation

One of the usual end products of a cost analysis is a learning curve or a series of learning curves graphically presented on logarithmic grids with an accompanying narrative. The data are based typically on varied sources with varying degrees of validity. For the practical cost analyst, the study is frequently the end-time result of a harsh deadline.

Under such circumstances, he is too far removed from the data sources to undertake a detailed investigation of the new unit cost which would result from a major change or from semi-finished materials, portions of whose costs are chargeable to a succeeding lot.

These investigations would require participation with an on-site auditor or a service plant representative.*

7.2 Responsibility of the Analyst

Notwithstanding the difficulty associated with such investigations, during the time span available, the cost analyst should reflect his uncertainty with appropriate footnotes on the graphs or discussion in the narrative. This will serve to alert the decision maker or reviewer to areas of further investigation which may be open to him; specifically, a re-evaluation of the data base through an extension of the study.

Rates of improvement are not always uniform during an entire production run, particularly if there is a long time period involved. The learning curve, therefore, does not show a constant slope. There may be two or more periods exhibiting different learning rates. The contributing cause for this phenomenon is frequently the rapid introduction of improved tooling and production methods during early production. For this reason, the learning curve is most fallible in low orders of quantity produced and generally exhibits slopes steeper than will be experienced later during relatively stable conditions.

Cost analysis projections should include geometrical considerations and statistics in combination with informed intuition. With comparatively few data points and limited time, it may be impossible to fit a log-linear line by the statistical regression analysis techniques. Lacking the standard error, the precise determination of an incorrect data point under such circumstances is largely a matter of conjecture. Nevertheless, there is usually a highest point and a lowest point. Either of these warrant investigation and very likely discussion in the narrative; the former is the telltale sign of a major engineering change order.

8. SUMMARY

The learning curve, as expressed by Equation 1 in the Appendix, is the best available relationship to connect direct labor cost or manhours with cumulative units produced. It is intuitively reasonable and has been tested in many industries and processes.

The learning curve appears in two forms. One form expresses the dependent variable as unit cost, the other as cumulative average cost. In relation to aircraft, these, in turn, may be expressed as

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* Defense Contract Audit Agency (DCAA) and the Defense Contract Administration Service (DCAS) respectively.

cost or cost per pound for either unit or cumulative average cost. Any of the above may be relevant but, for clarity, care should be exercised to identify which is under consideration.

Computational and statistical techniques present no difficulty since they are developed and discussed in an extensive literature.

The major difficulty associated with the application of the model is the identification of the parameters in the equation which frequently rest on an insecure data base. Rapid improvements are being made in the data base by the Department of Defense Cost Information Reports.

As a result of complex causes, the cost-quantity relationship expressed by the learning curve is always subject to a certain amount of variation. Some stable system of causes, however, is inherent in any learning curve. Variation within the stable pattern is inevitable. The reasons for variation outside the pattern may be discovered and described. Consequently, corrections may be made and explained to improve forecasting.

With a firm model, extrapolation becomes largely a mechanical process. Failing this and faced with a pressing deadline, the cost analyst must frequently resort to intuition. His function in this regard is to explain the unusual: Show why the learning rate is different from a similar predecessor product; show why there are unmatched theoretical first unit costs; show why a data point is unusually high or low. He should document his findings, to separate fact and mathematics from intuition and assumption.

APPENDIX

Learning Curve

1 $y = ax^b$

y cost or direct labor manhours per unit. In the case of airframes y is sometimes expressed as dollars or hours per lb.

2 $\text{Log } y = \text{Log } a + b \text{ Log } x$

3 $2^b = 1\text{-}p$

p the constant percentage reduction as cumulative units are doubled. (1-p) is the conventional method of recording the learning effect.

4 $$K = \left[\frac{L(1-b)}{N_1^{1-b} - N_2^{1-b}} \right]^{-\frac{1}{b}}$$

K algebraic lot mid point
N_2 first unit in lot minus $\frac{1}{2}$
N_1 last unit in lot plus $\frac{1}{2}$
L number of units in the lot
b a parameter. The exponent in the learning relationship

Regression Analysis

Since equation 2 is the log-linear transform of 1, the summation statistics used as substitutions into the regression equations must be transformed to logarithms (Σ xy to Σ log x·log y, Σ x Σ y to Σ log x·Σ log y, etc.)

For simplicity, the equations below are shown for the linear form

5 $$b = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$$

n the number of data points, usually the number of lots.

... exponent b in the log-linear form is the tangent of the ... ratios defined by the symbol $\Delta y/\Delta x$ measured on arithm... ...

$$a = \frac{\Sigma y - b \Sigma x}{n}$$

$$r^2 = \frac{[n \Sigma xy - \Sigma x \Sigma y]^2}{[n \Sigma x^2 - (\Sigma x)^2]\,[n \Sigma y^2 - (\Sigma y)^2]}$$

$$\sigma^2_{y \cdot x} = \frac{\Sigma z^2}{n-2}$$

$$= \frac{\Sigma y^2 - b \Sigma xy - a \Sigma y}{n-2}$$

z the residuals along the line of best fit.

...he denominator of 8 is adjusted from the general case of n... ... is the number of parameters in the regression situation. ...here are ... ...meters in Equation 1 giving n-2 degrees of freedom for th... unbiased ... of the standard error.

An abbreviated table for two levels of confidence is given below. In the table v = n-m. More elaborate tables are generall... available.

"t" TABLE

For Two-Sided Confidence Bands
About the Linear Regression Equation

degrees of freedom	confidence	
v	90%	95%
1	6.314	12.706
2	2.920	4.303
3	2.353	3.182
4	2.132	2.776
5	2.015	2.571
6	1.943	2.447
7	1.895	2.365
8	1.860	2.306
9	1.833	2.262
10	1.812	2.228

...alues in the body of the table are used as multipliers against the stan...ard error for given confidence levels.

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13. ABSTRACT

Cost estimates are sometimes poorly prepared because the learning curve is misunderstood or badly applied.

The learning curve as a tool, regression analysis as a curve fitting technique and various devices for adjusting learning data from practical process knowledge are addressed separately in a voluminous literature. This paper puts down the main framework of necessary concepts in the application of the learning curve with appropriate references to the literature.

Experience shows that, at some point in an analysis, the estimator is required to enter opinions because of lack of data, incomplete knowledge of the process or other causes beyond his control. The emphasis in this paper is to distinguish between mathematics and judgment; between calculation and intuition; putting cautions on the analyst to provide the reviewer with visibility as to where one ends and the other begins.

Topics include a description of the forms of the learning curve with distinctions among possible variables, various necessary calculations and conversions, fundamental concepts related to the location of a straight line in two dimensional space, factors which contribute to learning in industrial processes and adjustments for special circumstances.

The document was prepared as guidance to cost analysts in the US Army Materiel Command.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Cost Quantity Relationships						
Learning Curve						
Estimation						
Cost Analysis						
Judgment and Intuition, versus Computation and Mathematics						

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